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THE *GM* VALUES OF MIMAS AND TETHYS AND THE LIBRATION OF METHONE

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ABSTRACT

We have determined the *GM* of the Saturnian satellite Mimas from an analysis of its resonance with Tethys and with the newly discovered satellite Methone (*GM* is the product of the Newtonian constant of gravitation *G* and the satellite's mass *M*). Observations of the latter permit a factor of 5 improvement in our knowledge of Mimas's *GM*; its value is 2.504 ± 0.002 km^3 s^{-2}. Tethys's *GM* was originally found from the Mimas-Tethys resonance. We, however, estimate it using observations of its two Lagrangian satellites, Calypso and Telesto; its value is 41.200 ± 0.007 km^3 s^{-2}.

Key word: planets and satellites: individual (Mimas, Tethys)

1. INTRODUCTION

The mean motions of the Saturnian satellites Mimas and Tethys are in near 2 : 1 commensurability, resulting in a mean longitude libration with an ~72 yr period; the amplitudes of the libration are ~43° for Mimas and ~2° for Tethys. Kozai (1957) exploited the libration to find the *GM* values of the two satellites (*GM* is the product of the Newtonian constant of gravitation[1] *G* and the satellite's mass *M*) from about 100 yr (1846–1947) of astrometric observations. The frequency of the libration provided him with a measure of Tethys's *GM*, and with it known he obtained Mimas's *GM* from the ratios of the libration amplitudes. Applying the same procedure, the *GM*s were determined by Dourneau (1987) and Dourneau & Baratchart (1999) with observations from 1886 to 1985 and by Harper & Taylor (1993) with observations from 1874 to 1989. Tyler et al. (1982) estimated the *GM* of Tethys from the *Voyager 2* radio science data and used Kozai's mass ratio to infer Mimas's *GM*. As a part of the development of their dynamical theory for the motions of the Saturnian satellites, Vienne & Duriez (1995) determined the *GM*s of Mimas and Tethys from essentially the same data set as Harper & Taylor.

In preparation for the *Cassini* tour of the Saturnian system, we developed numerically integrated ephemerides for the Saturnian satellites (Jacobson 2004). Our satellite system contained not only the eight major satellites but also Phoebe and the Lagrangian satellites Helene, Telesto, and Calypso. Helene oscillates about the leading triangular libration point of Dione with a period of ~780 days. Telesto and Calypso oscillate about the Tethys leading and trailing libration points, respectively, with periods of ~660 days. We fitted the integration to astrometry from Earth-based observatories and the *Hubble Space Telescope*, spacecraft imaging from the *Voyager* spacecraft, and radiometric tracking of the *Voyager* and *Pioneer 11* spacecraft. Overall, our data arc spanned the 37 yr from 1966 to 2003. Adding the observations of the Telesto and Calypso librations to those of the Mimas libration led to a significant improvement in the determination of Tethys's *GM*. From that new *GM* of Tethys coupled with the high quality of the recent Mimas and Tethys astrometry, we were able to get a reasonably good estimate of Mimas's *GM* even though the data arc was only about half the Mimas-Tethys libration period.

Subsequent to the arrival of *Cassini* at Saturn, both the *Cassini* Imaging Science team and the Optical Navigation team began acquiring imaging observations of the known satellites using the *Cassini* Imaging Science Subsystem (Porco et al. 2004b). We continued our ephemeris development, adding this data as it became available (Jacobson et al. 2004, 2005a, 2005b). We also lengthened our Earth-based data arc by introducing the visual micrometer observations of the eight major satellites made at the US Naval Observatory during the years 1938–1947. No significant change in the estimate of Tethys's *GM* occurred when we included the micrometer data and the *Cassini* observations; however, the uncertainty in the estimate dropped by an order of magnitude. As a consequence of the longer data arc and the better known *GM* of Tethys, we found a small improvement in our knowledge of Mimas's *GM*. A historical summary of the *GM* determinations appears in Table 1.

2. ANALYSIS

Early in the *Cassini* tour of the Saturnian system, the *Cassini* Imaging Science team discovered the satellite Methone (S/2004 S1) orbiting between Mimas and Enceladus (Porco et al. 2004a, 2005). As with the other satellites, the orbit of Methone was modeled with a numerical integration of its equations of motion (Spitale et al. 2006). Because its small size (its diameter is ~3 km) allowed us to ignore Methone's gravitational effect on the other satellites, we integrated its orbit separately from those of the major satellites. The integration, however, included the gravitational perturbations due to those satellites, the Sun, and Jupiter and the asphericity of Saturn. Analysis of Methone's orbital motion uncovered a mean longitude resonance with Mimas having a period of ~450 days and an amplitude of ~5°. The argument of the resonance satisfies the relation $\theta = 15\lambda - 14\lambda' - \varpi$, where λ and ϖ are the mean longitude and longitude of periapsis

[1] Mohr & Taylor (2005) discuss the determination of *G* and recommend a value.

TABLE 1

SUMMARY OF *GM* DETERMINATIONS

Mimas[a]	Tethys[a]	Reference
2.50 ± 0.04	41.7 ± 0.8	Kozai (1957)
3.0 ± 0.4	50 ± 6	Tyler et al. (1982)
2.46 ± 0.08	41 ± 1	Dourneau (1987)
2.45 ± 0.04	40.8 ± 0.7	Harper & Taylor (1993)
2.40 ± 0.09	40.2 ± 0.5	Vienne & Duriez (1995)
2.47 ± 0.08	41.5 ± 1.2	Dourneau & Baratchart (1999)
2.55 ± 0.05	41.21 ± 0.08	Jacobson (2004)
2.55 ± 0.02	41.210 ± 0.009	Jacobson et al. (2004)
2.54 ± 0.02	41.210 ± 0.008	Jacobson et al. (2005a)
2.53 ± 0.01	41.210 ± 0.007	Jacobson et al. (2005b)
2.504 ± 0.002	41.200 ± 0.007	Current analysis

[a] *GM* in units of $km^3\ s^{-2}$.

of Methone and λ' is the mean longitude of Mimas. Figure 1 shows Methone's osculating mean longitude with its secular rate removed; the long-period resonance is clearly visible.

We found that our initial orbit did not fit the *Cassini* imaging observations as well as might be expected. This led us to suspect that there might be a problem with the modeling of Mimas's effect on Methone. At the time we were using the Mimas orbit and *GM* from Jacobson et al. (2005b) to perturb Methone.

In order to investigate the problem we added Methone to our general analysis of the Saturnian satellite system in the same manner as we have done with the Lagrangian satellites. Namely, we integrated Methone's orbit along with the orbits of the major and Lagrangian satellites and processed the Methone observations together with all of the other satellite observations. Including Methone in the general analysis had three advantages:

1. The variational equations describing changes in Methone's orbit due to changes in the gravity field and other satellite orbits were complete.
2. Estimated changes in Mimas's orbit or *GM* needed to improve Methone's orbit were forced to be consistent with the full observational set.
3. Both the Mimas-Tethys and Mimas-Methone resonances contributed directly to the Mimas *GM* determination.

The *GM*s found from the general analysis including Methone appear as the final entry in Table 1. There is a nearly 2.6 σ reduction in the Mimas *GM* value and a factor of 5 reduction in its uncertainty. It is interesting that our value agrees with the one Kozai (1957) obtained nearly 50 years ago; our uncertainty, however, is a factor of 20 smaller. Tethys's *GM* drops by slightly more than 1 σ, but its uncertainty is unchanged. Figure 2 displays the time history of the Methone sample and line residuals for the orbit based on the previous Mimas *GM*; Figure 3 shows the residuals from the current orbit. The change in the *GM* results in a dramatic improvement in the quality of the orbit fit.

Comparing the residual plots with Figure 1 shows that the observation arc covers a significant portion of the resonance period. Additional observations from the remainder of the prime and extended *Cassini* mission should lead to further improvements.

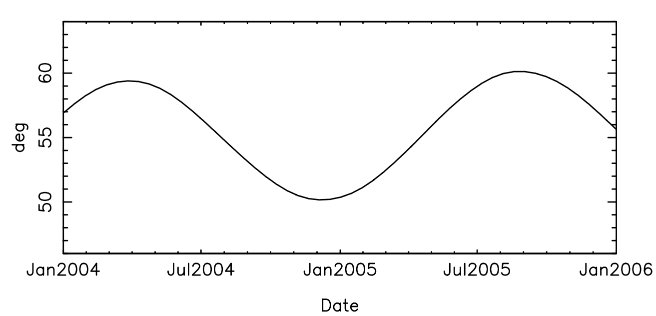

FIG. 1.—Methone mean longitude with secular rate removed.

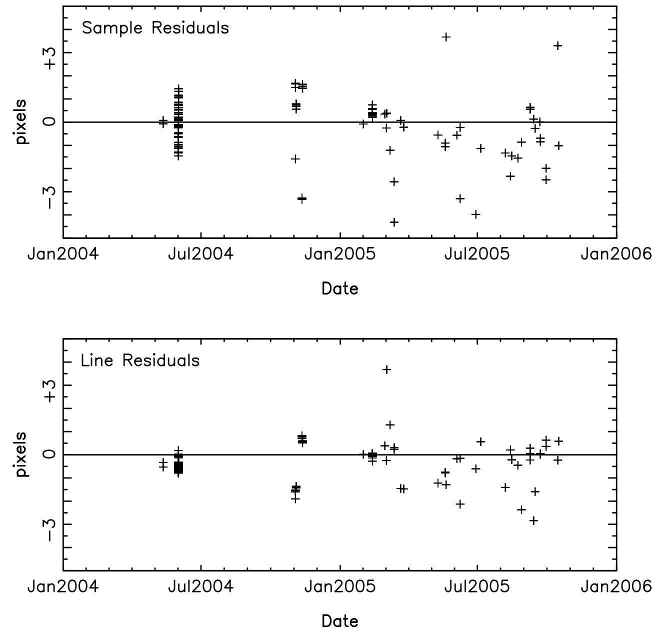

FIG. 2.—Initial Methone observation residuals.

3. CONCLUDING REMARKS

The *Cassini* spacecraft has had close encounters with all of the major Saturnian satellites with the exception of Mimas. By analyzing the spacecraft radiometric tracking data acquired during those encounters, the satellite *GM*s can be deduced from the satellite perturbations of the spacecraft trajectory. Improved values have been found for the *GM*s for Enceladus, Rhea, Hyperion,

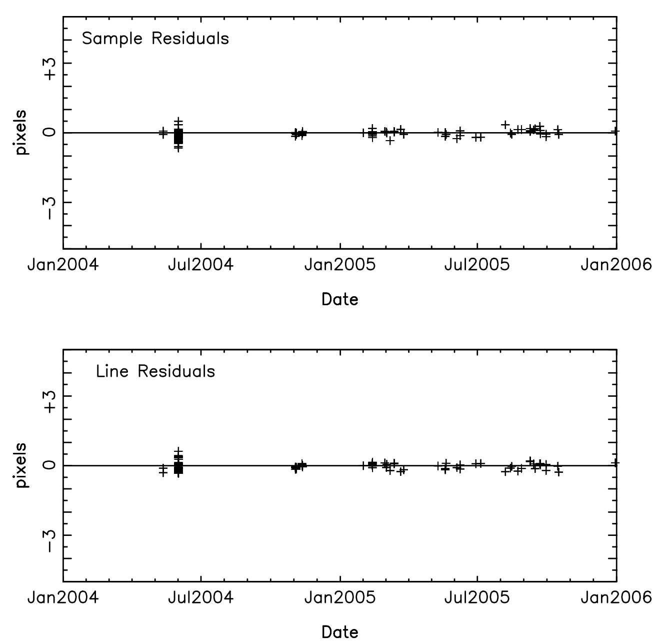

FIG. 3.—Current Methone observation residuals.

Iapetus, and Phoebe, and the uncertainty on Titan's *GM*, which was determined from the *Voyager 1* flyby, has been significantly reduced. The tracking data have also confirmed Tethys's and Dione's *GM*s, but the dominant source of information on their values remains the observations of the Lagrangian satellites. As there are no *Cassini* flybys of Mimas planned, there will be no radiometric determination of its *GM*. Through the discovery of Methone and the collection of imaging data of it, however, *Cassini* has provided a means to determine Mimas's *GM* to a level of accuracy exceeding that possible from Earth-based observations.

A detailed discussion of the Methone orbit can be found in Spitale et al. (2006). A publication on the overall gravity field of the Saturnian system is in preparation.

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